Induced drag and Vorticity
in Viscous and Inviscid flows

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Digital Twin: the key role of POST-PROCESSING

- Rarely addressed, post-processing is as crucial as other simulation stages, for a digital twin to provide relevant information.
- Aerodynamic designers interested in decomposing drag into physical contributions: viscous, wave and lift-induced drag components.
- Shape design based on numerical optimisation takes great advantage of drag breakdown information:
  - accurate drag prediction on relatively coarse meshes, removing part of the so-called "spurious drag".
  - identification of design variables with major influence on specific drag components.
- Deeper understanding on the genesis of different aerodynamic force contributions.

State of the art:

- Available commercial and/or open-source flow solvers/post-processors do not provide information on physical drag contributions.
- Drag decomposition methods now routinely used in Commercial Aviation industry, supported by Research Organizations providing post-processing software.

Main objectives

starting from two simple questions:

How to compute lift-induced drag?

- For a planar wing, *Prandtl* formula $C_{D,i} = \frac{C_L^2}{\pi AR \cdot e}$ can be used, but...
- ...the *Oswald*'s factor is not known a priori.
- ...in the case of zero lift coefficient, *Prandtl* formula gives zero induced drag, which is not generally correct in case of non-zero wing loading! (Torenbeek)
- We need other (far-field) methods.

Is lift-induced drag zero in a steady two-dimensional flow?

- Available far-field definitions of lift-induced drag (e.g. *Maskell*'s formula) do not mathematically ensure zero drag in 2D flows.
- However, no trailing vorticity suggests no lift-induced drag...

We will focus on analysing available drag definitions, their behaviour and limitations.

Drag Definitions

- **REVERSIBLE drag**: drag generated in isentropic flow.
- **IRREVERSIBLE drag**: drag related to entropy production.

- **LIFT-INDUCED drag**: drag related to global (or local) lift generation and associated trailing vorticity.
- **PARASITE drag**: drag not directly related to global (or local) lift generation.

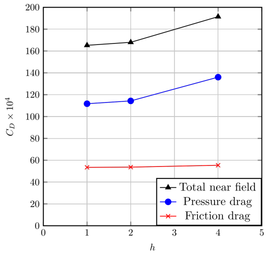
«An ambition which will have to wait is a rigorous definition of induced drag in viscous flows» Spalart (JFM, 2008).

Near Field vs. Far-Field Force Methods

Integral Balance of Linear Momentum

Near Field (NF) aerodynamic force:

$$\boldsymbol{F}_{nf} = \int_{S_B} \left(p\boldsymbol{n} - \underline{\underline{\tau}}_v \cdot \boldsymbol{n} \right) \mathrm{d}S$$



NACA 0012, $M_\infty = 0.7$, $Re_\infty = 9.0 \times 10^6$, $\alpha = 3.25°$.
SU2 RANS solution, SA turb. model.
$h = 1,\ 2,\ 4$: $256,\ 128,\ 64$ cells around airfoil. NASA C-type grid.

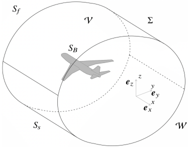
Computed drag converges as mesh size h is reduced, but:

- Accurate drag prediction only on extremely fine grids.
- Drag breakdown in pressure and friction components only.

An alternative is: **Far Field** (FF) aerodynamic force:

$$\boldsymbol{F}_{ff} = -\int_{\Sigma} \left[\rho \boldsymbol{V}\boldsymbol{V} \cdot \boldsymbol{n} + (p - p_\infty)\,\boldsymbol{n} - \underline{\underline{\tau}}_v \cdot \boldsymbol{n} \right] \mathrm{d}S$$

- Identification of physical drag contributions.
- deeper understanding on the genesis of aerodynamic force.



University of Naples (UNINA) and CIRA started studies on 1998 (EU-funded AIRDATA project).

Available force methods for drag decomposition

and associated **operative drag definitions**

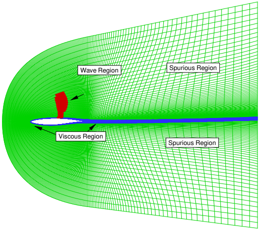
- **THERMODYNAMIC** methods:
  Allow estimating irreversible drag only.
  Reversible drag is indirectly obtained by difference with near-field total drag (not linked to any field data).
  No information on lift.
- **CLASSICAL** far-field methods:
  Available formulae for lift, lift-induced drag and parasite drag (Kutta-Joukowski theorem, Maskell's and Betz's formulae).
- **VORTICAL** methods:
  Definition of both lift and drag, with total drag decomposition in lift-induced and parasite contributions (links with classical far-field formulae available).

Thermodynamic Methods

$$\frac{V}{V_\infty} = f\left(\frac{\Delta p}{p_\infty}, \frac{\Delta s}{R}, \frac{\Delta H}{V_\infty^2}\right)$$

plugging into general far-field drag expression, neglecting pressure variations and viscous stresses, and shifting to divergence form:

$$D_{ff} = -V_\infty \int_{\mathcal{V}} \boldsymbol{\nabla} \cdot \left[\rho \hat{v_x} f\left(\frac{\Delta s}{R}, \frac{\Delta H}{V_\infty^2}\right) \boldsymbol{V}\right] \mathrm{d}\mathcal{V}$$



Paparone and Tognaccini (AIAA J., 2003): $\Delta H = 0$.
Destarac and Van Der Vooren (AS&T, 2004): $\hat{v_x} = 0$.

- Obtained breakdown in viscous and wave drag.
- Spurious drag removal, improving accuracy.
- Flow regions selected using proper sensors.

$$D_v = -V_\infty \int_{\mathcal{V}_v} \boldsymbol{\nabla} \cdot (\ldots)\, \mathrm{d}\mathcal{V}$$

$$D_w = -V_\infty \int_{\mathcal{V}_w} \boldsymbol{\nabla} \cdot (\ldots)\, \mathrm{d}\mathcal{V}$$

$$D_{sp} = -V_\infty \int_{\mathcal{V}_{sp}} \boldsymbol{\nabla} \cdot (\ldots)\, \mathrm{d}\mathcal{V}$$

Entropy and total enthalpy variations only due to dissipative effects in steady un-powered conditions (irreversible drag).

Classical far-field methods (1/3)

After some manipulation, the general FF formula is expressed as ($\delta\boldsymbol{V} = \boldsymbol{V} - \boldsymbol{V}_\infty$):

$$\boldsymbol{F}_{ff} = \int_\Sigma \left[(p_{t,\infty} - p_t)\boldsymbol{n} + \frac{1}{2}V_\infty^2(\rho - \rho_\infty)\boldsymbol{n} + \underline{\underline{\tau}}_v \cdot \boldsymbol{n} \right] \mathrm{dS} + \\ + \int_\Sigma \left[-\rho\delta\boldsymbol{V}\delta\boldsymbol{V}\cdot\boldsymbol{n} + \frac{1}{2}\rho(\delta V^2)\boldsymbol{n} + \boldsymbol{V}\infty \times \boldsymbol{n} \times \rho\delta\boldsymbol{V} \right] \mathrm{dS}$$

In **homoentropic** and **incompressible** flows, the first integral is zero, and the second one provides lift and lift-induced drag:

$$\boldsymbol{L} = \underbrace{\rho_\infty \boldsymbol{V}\infty \times \int_\Sigma \boldsymbol{n} \times \delta\boldsymbol{V}\mathrm{dS} + \rho_\infty \int_\Sigma \left[\frac{1}{2}(\delta V^2)n_z - w\delta\boldsymbol{V}\cdot\boldsymbol{n} \right] \mathrm{dS}\boldsymbol{e}_z}_{\text{3D Kutta-Joukowski theorem with finite-domain correction term}} \triangleq \boldsymbol{F}_{KJ}$$

$$D = \underbrace{\rho_\infty \int_\Sigma (\frac{1}{2}u^2 n_x - u\delta\boldsymbol{V}\cdot\boldsymbol{n})\mathrm{dS} + \rho_\infty \int_\Sigma \frac{1}{2}(v^2 + w^2)n_x \mathrm{dS}}_{\text{Maskell's formula (LIFT-INDUCED DRAG)}} \triangleq \boldsymbol{F}_{MSK} \cdot \boldsymbol{e_x}$$

In these conditions, there's no entropy production, therefore Maskell's formula provides an exact definition of the reversible drag.

Classical far-field methods (2/3)

In general **viscous** and **compressible** flows, supposing that:

- the upstream and lateral portions of Σ are not too close to the body and the flow is not supersonic.
- the downstream portion of Σ is orthogonal to the free-stream velocity $\boldsymbol{V}_\infty$.
- viscous stresses on Σ can be neglected.

the first integral in F_{ff} has no lift component. It represents a *parasite* drag.

The second integral provides lift, and its streamwise component is assumed as lift-induced drag definition:

$$D = \underbrace{\int_\Sigma \left[(p_{t,\infty} - p_t)\boldsymbol{n} + \frac{1}{2}V_\infty^2(\rho - \rho_\infty)\boldsymbol{n} + \underline{\underline{\tau}}_v \cdot \boldsymbol{n} \right] \mathrm{dS} \cdot \boldsymbol{e}_x}_{\boldsymbol{F}_{BETZ} \cdot \boldsymbol{e_x}\text{: Compressible Betz's formula (PARASITE DRAG)}} +$$

$$+ \underbrace{\int_\Sigma (\frac{1}{2}\rho u^2 n_x - \rho u \delta \boldsymbol{V} \cdot \boldsymbol{n})\mathrm{dS} + \int_\Sigma \frac{1}{2}\rho(v^2 + w^2)n_x \mathrm{dS}}_{\text{Extended Maskell's formula (LIFT-INDUCED DRAG)}}$$

These definitions are however subject to the above assumptions. A general choice of Σ would not justify the appellation of *parasite* drag for the first integral.

Classical far-field methods (3/3)

$$D = \underbrace{\int_{\Sigma} \left[(p_{t,\infty} - p_t)\boldsymbol{n} + \frac{1}{2} V_{\infty}^2 (\rho - \rho_{\infty})\boldsymbol{n} + \underline{\underline{\tau}}_v \cdot \boldsymbol{n} \right] \mathrm{dS} \cdot \boldsymbol{e}_x}_{\text{Compressible Betz's formula (PARASITE DRAG)}}$$

$$+ \underbrace{\int_{\Sigma} (\frac{1}{2}\rho u^2 n_x - \rho u \delta \boldsymbol{V} \cdot \boldsymbol{n}) \mathrm{dS} + \int_{\Sigma} \frac{1}{2}\rho (v^2 + w^2) n_x \mathrm{dS}}_{\text{Extended Maskell's formula (LIFT-INDUCED DRAG)}}$$

Additionally, a total drag decomposition in lift-induced drag and parasite contributions using above definitions, is not always consistent with drag breakdown in reversible and irreversible components.

Consider, for example, a non-dissipative subsonic flow regime:

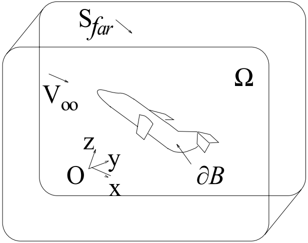
- In this case, there's no entropy production, then the whole drag is reversible drag.
- Nevertheless, parasite drag as defined above is non-zero in compressible flows.
- $\rightarrow$ lift-induced drag as defined above does not coincide with total reversible drag.

We will further analyse breakdown consistency after introducing vortical methods...

Vortical far-field methods (Lamb vector-based: $\boldsymbol{\ell} = \boldsymbol{\omega} \times \boldsymbol{V}$)

Exact expression of the aerodynamic force[a]:

$$\boldsymbol{F} = \boldsymbol{F}_{\boldsymbol{\ell}} + \boldsymbol{F}_{m_\rho} + \boldsymbol{F}_S + \boldsymbol{\Delta}_\mu$$



Vortex force

$$\boldsymbol{F}_{\boldsymbol{\ell}} = -\int_\Omega \rho \boldsymbol{\ell} \mathrm{d}V$$

Compressibility correction

$$\boldsymbol{F}_{m_\rho} = -\int_\Omega \boldsymbol{r} \times \left[\nabla\rho \times \nabla\left(\frac{V^2}{2}\right)\right] \mathrm{d}\Omega$$

Outer vorticity contribution

$$\boldsymbol{F}_S = -\int_{S_{\mathrm{far}}} \boldsymbol{r} \times \left[\boldsymbol{n} \times (\rho\boldsymbol{\ell})\right] \mathrm{dS}$$

Terms explicitly depending on viscosity

$$\boldsymbol{\Delta}_\mu = \int_{S_{\mathrm{far}}} \boldsymbol{r} \times \left[\boldsymbol{n} \times \nabla \cdot \underline{\underline{\tau}}_v\right] \mathrm{dS} + \int_{S_{\mathrm{far}}} \underline{\underline{\tau}}_v \cdot \boldsymbol{n} \mathrm{dS}$$

[a] Wu et al. (JFM 2007), Mele & Tognaccini (PoF 2014)

Link between classical and vortical formulations

Fournis et al. (AIAA 2021-2554) derived two notable identities linking vortical force theory to classical far-field formula:

$$\boldsymbol{F} = \overbrace{\underbrace{\boldsymbol{F}_{\ell} + \boldsymbol{F}_{m_\rho} - \boldsymbol{F}_{\nabla\rho}}_{= \boldsymbol{F}_{KJ} + \boldsymbol{F}_{MSK}}}^{\substack{\text{lift} \\ + \\ \text{lift-induced drag}}} + \overbrace{\underbrace{\boldsymbol{F}_{\nabla\rho} + \boldsymbol{F}_S + \boldsymbol{\Delta}_\mu}_{= \boldsymbol{F}_{BETZ}}}^{\text{parasite drag}}$$

where $\boldsymbol{F}_{\nabla\rho}$ is an additional compressibility correction, vanishing as $S_{far} \to \infty$.

Therefore:

- The same considerations done for classical lift-induced and parasite drag definitions, can be translated here to the vortical formulation.
- **The vortical formulation allows expressing lift and lift-induced drag as volume integrals (mid-field approach), identifying local sources of the force**.
- The total force formula is independent of the control volume size.
- Is it the same for this decomposition?

Relationship between ind./par. and rev./irr. decompositions

The relationship between **lift-induced/parasite** drag definitions and **reversible/irreversible** contributions is first analyzed considering three limit conditions:

3D inviscid subsonic flows — reversible drag only

- Is the defined **induced drag providing the whole drag?**

  Yes, as the Lamb vector is zero in the free-vortical wake ($\boldsymbol{F}_S = 0$) and the flow is inviscid ($\boldsymbol{\Delta}_\mu = 0$), provided S_{far} is not too close to the body ($\boldsymbol{F}_{\boldsymbol{\nabla}\rho} \approx 0$)

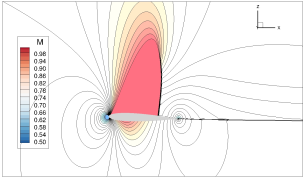
2D viscous subsonic flows — irreversible (viscous) drag only

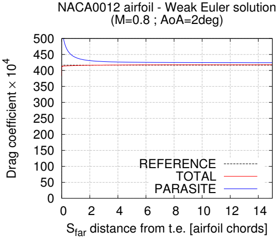
- Is the defined **parasite drag providing the whole** (viscous) **drag?**
- Kang et al. (AIAA J., 2019) numerically observed non-zero (negative) *spurious* induced drag values essentially depending on the Reynolds number of the simulation, using S_{far} close to the body.
- Kang's results were confirmed by a recent study (Minervino and Tognaccini, AIAA SciTech 2023).

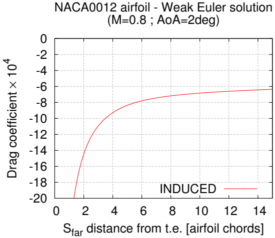
Relationship between ind./par. and rev./irr. decompositions

2D inviscid transonic flows irreversible (wave) drag only

- Is the defined **parasite drag providing the whole** (wave) **drag?**



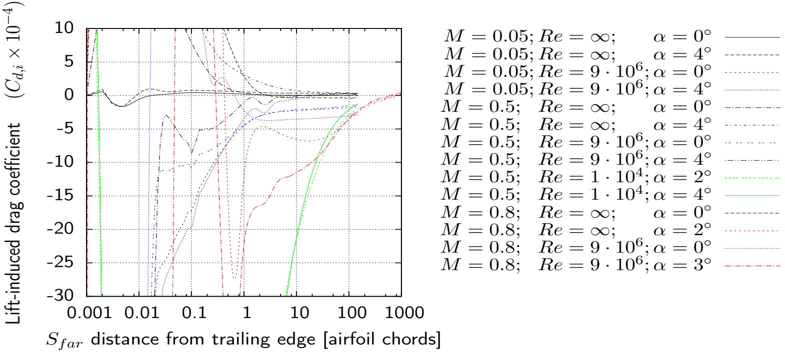




- Parasite drag slightly larger than total wave drag because of negative lift-induced drag, unless S_{far} is chosen sufficiently far from the body skin.
- Theoretical argumentation provided in a recent study (Minervino and Tognaccini, AIAA SciTech 2023).

Numerical evidence

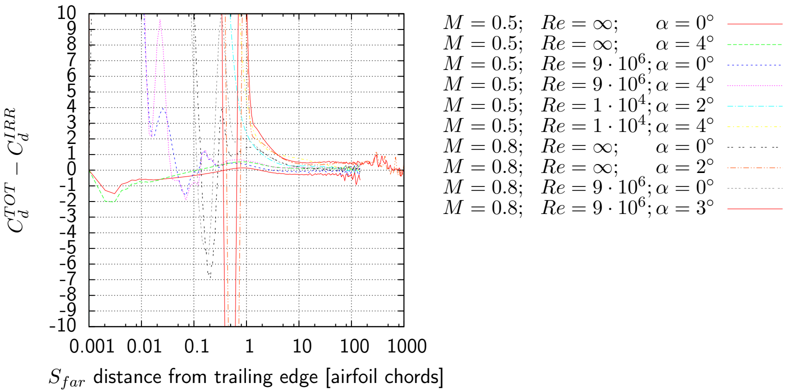
Lift-induced drag from different 2D flow-fields around NACA0012 airfoil



- in most of simulated cases (black curves), induced drag rapidly approaches zero.
- in stiff numerical simulations (blue curves), transonic asymmetric cases (red curves) or low Reynolds number conditions (green curves), the convergence of lift-induced drag to the (null) reversible drag is much slower.

Numerical evidence

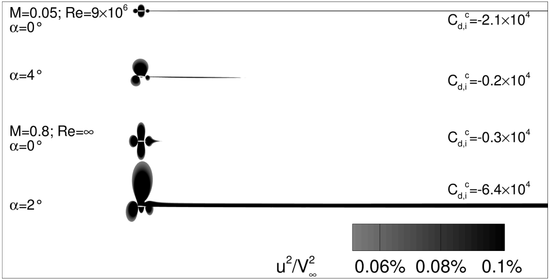
Irreversible nature of computed total drag (NACA0012 airfoil)



- A comparison of total drag vs. computed irreversible drag (D&VdV method) shows that indeed the whole drag is of irreversible nature.
- Convergence of computed irreversible drag to total drag is very fast.

Origin of negative lift-induced drag

Negative lift-induced drag related to streamwise velocity perturbation (u):



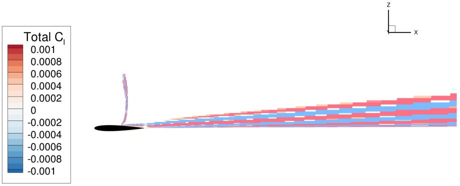
- Vortical expression of induced drag related to Maskell's formula (Fournis et al., AIAA 2021-2554):

$$D_{\mathrm{MSK}} = \frac{1}{2}\int_W \rho\left(v^2 + w^2 - u^2\right)\mathrm{d}S$$

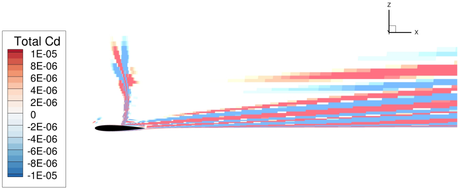
- Negative lift-induced drag allowed by the formula (through $-u^2$ term) and realizable at least in (idealized) two-dimensional flows.

Total lift and drag

NACA0012 airfoil - RANS solution ($M = 0.8, Re = 9 \times 10^6, \alpha = 3°$)



Total lift coeff.



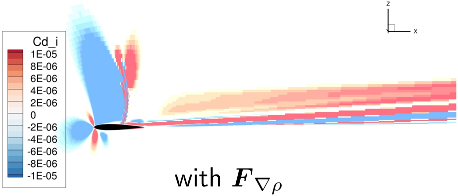
Total drag coeff.

The aerodynamic force is only generated in rotational flow regions where the Lamb vector is non-zero

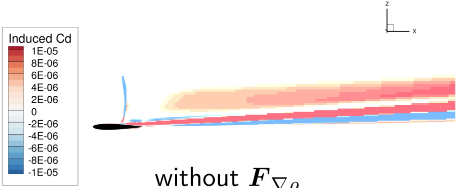
(generalized *Kutta-Joukowski* theorem).

Lift-induced drag

NACA0012 airfoil - RANS solution $(M = 0.8, Re = 9 \times 10^6, \alpha = 3^\circ)$



with $\boldsymbol{F}_{\nabla\rho}$



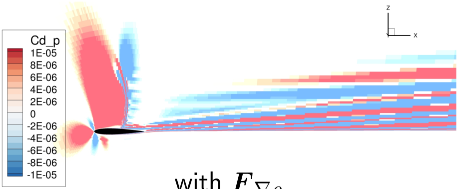
without $\boldsymbol{F}_{\nabla\rho}$

$$D_i = \left(\boldsymbol{F}_{\boldsymbol{\ell}} + \boldsymbol{F}_{\boldsymbol{m}_\rho} - \boldsymbol{F}_{\boldsymbol{\nabla}\rho}\right) \cdot \boldsymbol{e}_x$$

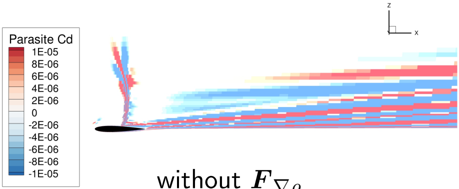
- $\boldsymbol{F}_{\nabla\rho}$ provides zero contribution when integrated on large control volumes: it can be removed from the contour plot to highlight regions effectively contributing to non-zero induced drag (in 3D).
- In 2D, even if lift-induced drag is zero over the whole flow-field, a local production is here visible (also in absence of streamline vortices).
- In 3D configurations, local production of lift-induced drag does NOT occur in the region of trailing vortices (zero Lamb vector).

Parasite drag

NACA0012 airfoil - RANS solution ($M = 0.8, Re = 9 \times 10^6, \alpha = 3°$)



with $\boldsymbol{F}_{\nabla\rho}$



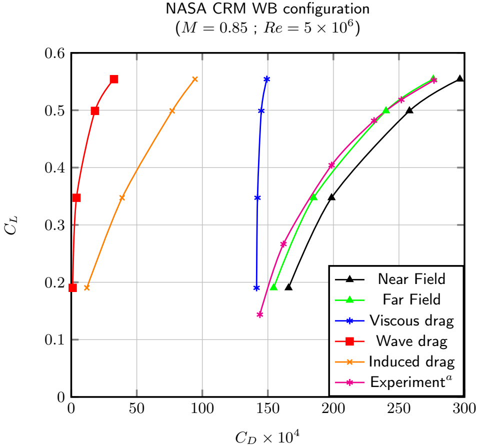
without $\boldsymbol{F}_{\nabla\rho}$

$$D_p = \left(\boldsymbol{F}_{\boldsymbol{\nabla}\rho} + \boldsymbol{F}_S + \cancel{\boldsymbol{\Delta}_\mu}\right) \cdot \boldsymbol{e}_x$$

- $\boldsymbol{F}_{\nabla\rho}$ provides zero contribution when integrated on large control volumes: it can be removed from the contour plot to highlight regions effectively contributing to non-zero parasite drag.
- $\boldsymbol{\Delta}_\mu$ contribution is negligible in high Reynolds number flows (Marongiu and Tognaccini, AIAA J., 2010).

An example of **thermodynamic** breakdown

Transonic wing-body configuration

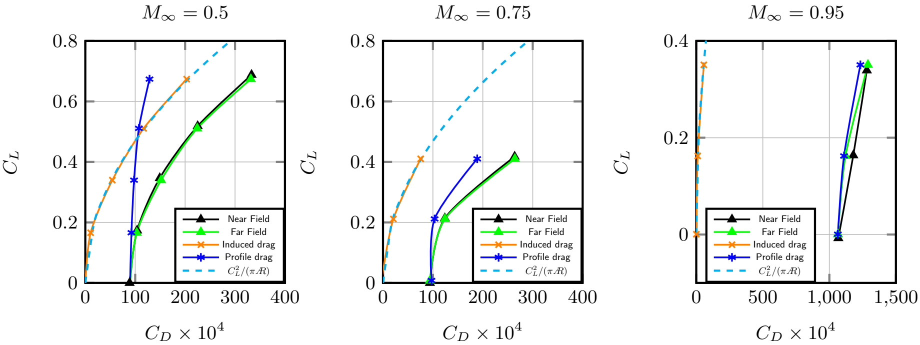


- On this coarse grid near field lift-drag polar far from experiment.
- Identification of spurious drag dramatically improves agreement with experiment.
- Obtained breakdown in viscous, wave and induced components.
- Industry is now routinely using thermodynamic methods.

[a] NASA Ames 11ft wind-tunnel.

An example of **vorticity-based** breakdown

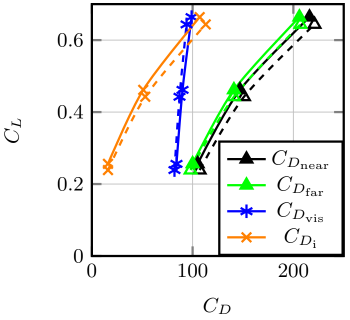
Elliptical wing in subsonic and transonic flow ($Re_\infty = 3 \cdot 10^6$)

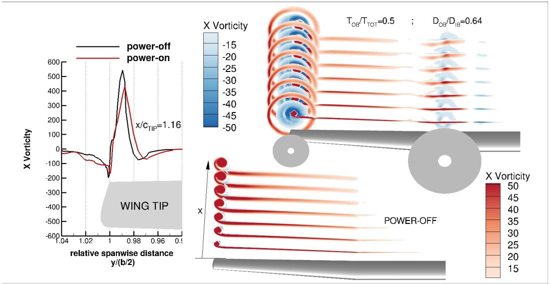


- Vortex-force definition of lift-induced drag in agreement with celebrated Prandtl's formula.
- Prandtl's formula correctly predicts lift-induced drag even in high transonic flow!

Distributed Electric Propulsion $(M_\infty = 0.48,\ Re_\infty = 16.6 \times 10^6)$

- Possibility to compare **Prop-ON** (solid) and **Prop-OFF** (dashed) conditions[b].
- Lift increased and total drag decreased by DEP.
- Very small increase of viscous drag in Prop-ON conditions.
- Increased Prop-ON span efficiency due to reduction of tip-vortex intensity.
- Thrust can be computed by vortex-force integral, Russo et al. (AIAA 2018-3967).





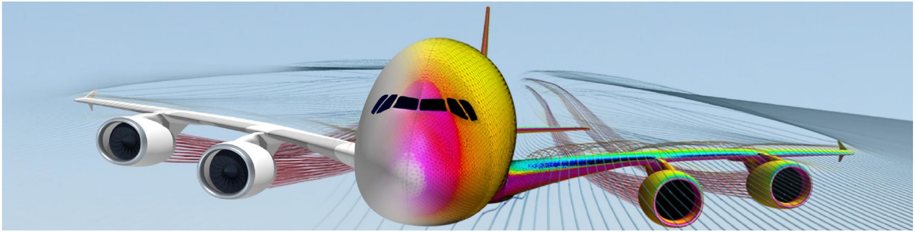
$\alpha = 0.0°$

[a] Minervino et al., *Drag Reduction by Wingtip-Mounted Propellers in Distributed Propulsion Configurations*, MDPI Fluids journal (open access) 2022, 7(7), 212, DOI: 10.3390/fluids7070212

Concluding Remarks and Upcoming Research activities

- The relationship between lift-induced/parasite drag decomposition and reversible/irreversible drag contributions has been discussed.
- Consistency is guaranteed on very large control volumes ($S_{far} \to \infty$).
- On control volumes of moderate size, non-zero lift-induced drag is obtained, while thermodynamic estimates of the irreversible drag still provide the whole drag.
- The observed non-zero lift-induced drag is related to the streamwise velocity perturbation, which role in the lift-induced drag formula is still ambiguous.
- The presence of a negative induced drag contribution, vanishing very slowly as $S_{far} \to \infty$, creates practical difficulties in 3D numerical applications.

- The Theoretical and Applied Aerodynamic Research Group at UNINA, together with CIRA, is actually working on aerodynamic force analysis and decomposition in the unsteady regime (foreseen applications to rotary wing analyses).



Thanks for your kind attention !

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