Robust Aerodynamic Design of a Supersonic Wing-Body for Natural Laminar Flow

Domenico Quagliarella, Emiliano Iuliano
Italian Aerospace Research Centre
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CIRA
Italian Aerospace Research Centre

FLEXIBLE ENGINEERING
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Università di Roma
Tor Vergata

Overview

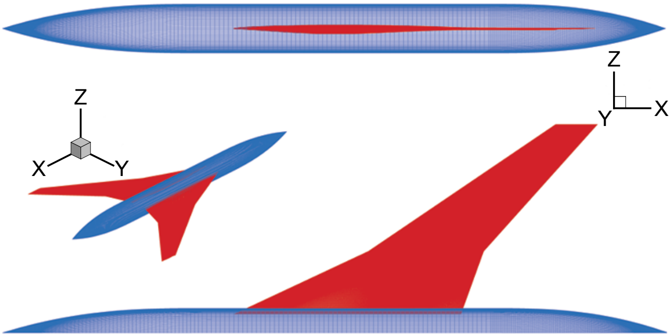
Summary I

- A robust optimization task was carried out, related to the Natural laminar Flow (NLF) design of a supersonic business jet
- The reference configuration is the deterministically optimized wingbody shape produced by CIRA within the SUPERTRAC EU project
- The objective is to obtain a new reliable design which offers a better behavior with respect to the uncertainties in operating conditions, wing-body shape and numerical performance prediction models
- The results presented here deal with the robust optimization of the baseline configuration with respect to uncertainties in the wing shape

Summary II

- The optimization framework here introduced is based on the Value-at-risk (VaR) risk measure, also called quantile, and it was originally introduced in the area of financial engineering.
- Very coarse VaR estimations are used in the optimization process and the bootstrap computational statistics technique is used to get an estimate of the standard error on of the risk function.

UMRIDA UQ Database Test Case IC-08



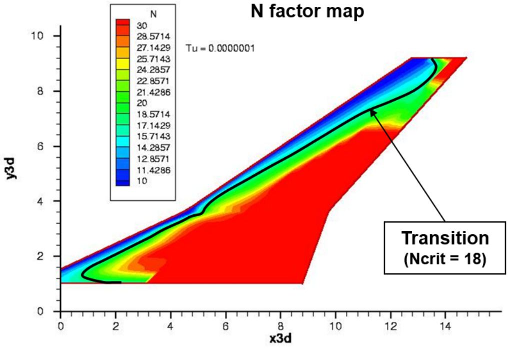
Wing body geometric features

Parameters	*Values*
Inboard sweep	65
Outboard sweep	56
Semi-span length	9.35 m
Aspect ratio	3.5
Wing area	50 m^2

Design flow conditions

Parameters	*Values*
Mach	1.6
Reynolds	51.8×10^6
L_{ref}	6.27 m
AOA	3.65
C_L	0.182

Optimization Problem



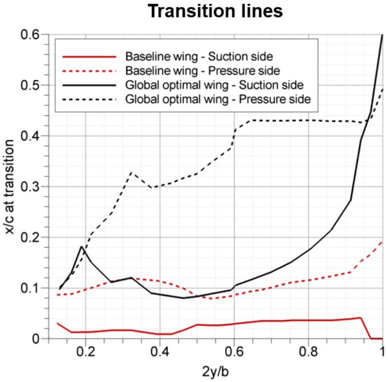


Figure from reference [1].

- Increase the extension of the laminar flow region
- Keep favorable values for the global aerodynamic coefficients

Optimization Problem Mathematical Formulation I

The objective function is defined as

$$G = K \frac{C_D + C_{D,M} + C_{D,L}}{C_L} \frac{\widetilde{C}_L}{\widetilde{C}_D} + (1-K)\frac{\Delta x_{\mathrm{lam}}}{\widetilde{\Delta x}_{\mathrm{lam}}} + \ell P\left(1 - \frac{ler}{\widetilde{ler}}\right) + tP\left(1 - \frac{tea}{\widetilde{tea}}\right)$$

with contribution to drag due to trim

$$C_{D,M} = \max[0, 0.05(\widetilde{C}_M - C_M)] \qquad C_{D,L} = \max[0, 1.0(\widetilde{C}_L - C_L)]$$

with $\widetilde{C}_L$ and $\widetilde{\Delta x}_{\mathrm{lam}}$ lift coefficient and the laminar extension indicator related to the baseline, K, ℓ, and t weighting coefficients (here $K = 0.25$, $\ell = 100$, and $t = 100$), and P quadratic penalty is activated if its argument is positive

Optimization Problem Mathematical Formulation II

Δx_{lam} estimates transition and laminar separation position on the whole wing

$$\Delta x_{\mathrm{lam}} = \sum_{i=1}^{n} (\Delta x_{\mathrm{lu}} + \Delta x_{\mathrm{ll}} + \Delta x_{\mathrm{su}} + \Delta x_{\mathrm{sl}})$$

where

$$\Delta x_{\mathrm{lu}} = \max\left(0, X^i_{\mathrm{tr}} - \overline{X}^i_{\mathrm{tr}}\right)_{\mathrm{upper}}$$
$$\Delta x_{\mathrm{ll}} = \max\left(0, X^i_{\mathrm{tr}} - \overline{X}^i_{\mathrm{tr}}\right)_{\mathrm{lower}}$$
$$\Delta x_{\mathrm{su}} = \max\left(0, X^i_{\mathrm{sep}} - \overline{X}^i_{\mathrm{sep}}\right)_{\mathrm{upper}}$$
$$\Delta x_{\mathrm{sl}} = \max\left(0, X^i_{\mathrm{sep}} - \overline{X}^i_{\mathrm{sep}}\right)_{\mathrm{lower}}$$

$\overline{X}^i_{\mathrm{tr}}$ and $\overline{X}^i_{\mathrm{sep}}$ are the computed values of transition and separation point at span section i, X^i_{tr} and X^i_{sep} are the desired values

Optimization Problem Summary

Design variables	
Wing section shape	User choice
Design point	
Mach number	1.6
Reynolds number	51 millions
Reference chord	6.27 [m]
Altitude	44000 [ft]
Lift coefficient	0.182
Design constraints	
Lift coefficient	$C_L \geq 0.180$
Pitching moment	$C_M \geq -0.05$
Trailing edge angle	$tea \geq \tilde{tea} = 0.050$ [rad]
Leading edge radius	$lea \geq \tilde{lea} = 0.0020$ [m]
Laminar extent — Suction side	$\overline{X_{\mathrm{tr}}/c} = 0.35$
Laminar extent — Pressure side	$\overline{X_{\mathrm{tr}}/c} = 0.45$
Laminar separation	$\overline{X_{\mathrm{sep}}/c} = 0.60$
Objective	
$G(C_L, C_D, C_M, ler, tea, \Delta x_{\mathrm{lam}})$	To be minimized

Computational Model Chain

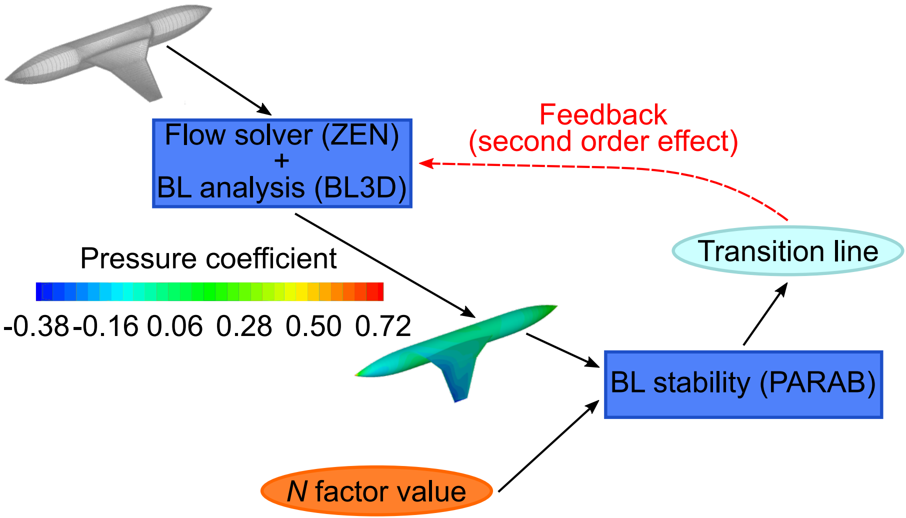


Figure from reference [2].

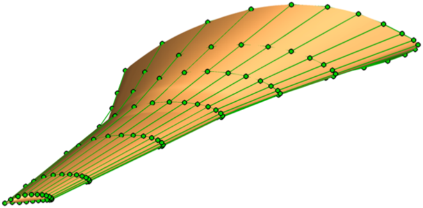
Uncertainty Sources

- **Geometrical uncertainties** Natural laminar flow is mostly sensitive to the shape of the leading edge region. This is due to its effect on pressure coefficient gradient which, in turn, is one of the factors that have most influence on the transition
- **Operational uncertainties** Operational uncertainties are here the classical one related to Mach number and lift coefficient (C_L)
- **Model uncertainties (epistemic)** One of the challenges that have to be faced when approaching the numerical design of natural laminar flow wings is the reliable estimation of the point of transition from laminar to turbulent flow. A significant uncertainty in the determination of transition location is inherent to the methods for numerical transition prediction and in particular to the e^N method, and to the related N_{critical} factor

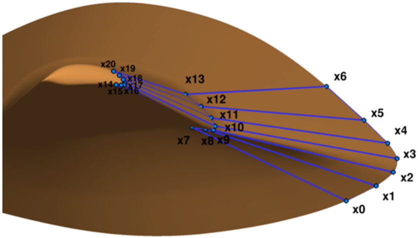
Currently, only geometrical uncertainties are considered in the design loop

NURBS Based Shape Parameterization

NURBS Control Points to control the whole wing shape (21 x 7)



Active NURBS Control Points to control local leading edge shape modifications (7 x 3)



- A grid of 21 x 7 NURBS CPs is defined on the whole wing surface
- A total number of 21 CPs is used to control the wing leading edge shape
  - 7 CPs streamwise, 3rd order basis functions
  - 3 CPs spanwise, 2nd order basis functions
- Design variables are the vertical displacements of the CPs

Uncertainty in Shape Description

- The uncertainty in the description of the wing shape is represented by a uniform random perturbation η_i that is added to the current NURBS control point position
- 21 design variables and, consequently, 21 shape perturbation parameters are considered in this problem.

design variable	lower bound	upper bound	random variable	lower bound	upper bound	distribution
y_0	-35	-15	η_0	-1	1	UNIFORM
y_1	-30	-10	η_1	-1	1	UNIFORM
y_2	-25	-5	η_2	-1	1	UNIFORM
y_3	-2	3.5	η_3	-0.275	0.275	UNIFORM
y_4	30	40	η_4	-0.5	0.5	UNIFORM
y_5	50	90	η_5	-2	2	UNIFORM
y_6	90	160	η_6	-3.5	3.5	UNIFORM
y_7	0	20	η_7	-1	1	UNIFORM
y_8	0	20	η_8	-1	1	UNIFORM
y_9	0	20	η_9	-1	1	UNIFORM
y_{10}	-2	3.5	η_{10}	-0.275	0.275	UNIFORM
y_{11}	5	30	η_{11}	-1.25	1.25	UNIFORM
y_{12}	20	50	η_{12}	-1.5	1.5	UNIFORM
y_{13}	40	80	η_{13}	-2	2	UNIFORM
y_{14}	-20	10	η_{14}	-1.5	1.5	UNIFORM
y_{15}	-20	10	η_{15}	-1.5	1.5	UNIFORM
y_{16}	-20	10	η_{16}	-1.5	1.5	UNIFORM
y_{17}	-2	3.5	η_{17}	-0.275	0.275	UNIFORM
y_{18}	5	30	η_{18}	-1.25	1.25	UNIFORM
y_{19}	10	30	η_{19}	-1	1	UNIFORM
y_{20}	15	50	η_{20}	-1.75	1.75	UNIFORM

Risk measure

- In optimization under uncertainty, inevitably, we have to deal with random events, modeled by random variables
- The first thing to do to treat the problem from a mathematical point of view, is to agree on a way to measure the risk $\mathcal{R}(X)$
- and on a level of risk that we consider acceptable, bearing in mind that there will inevitably be adverse events

$$\mathcal{R}(X) \leq C$$

- If the random variables representative of the costs depend on a deterministic decision vector, we are led naturally to:

$$\min_{x \in S \subseteq \mathbb{R}^n} \mathcal{R}_0(X_0(x)) \text{ s. to: } \mathcal{R}_i(X_i(x)) \leq c_i, \; i = 1, \ldots, m$$

Cumulative Distribution Function

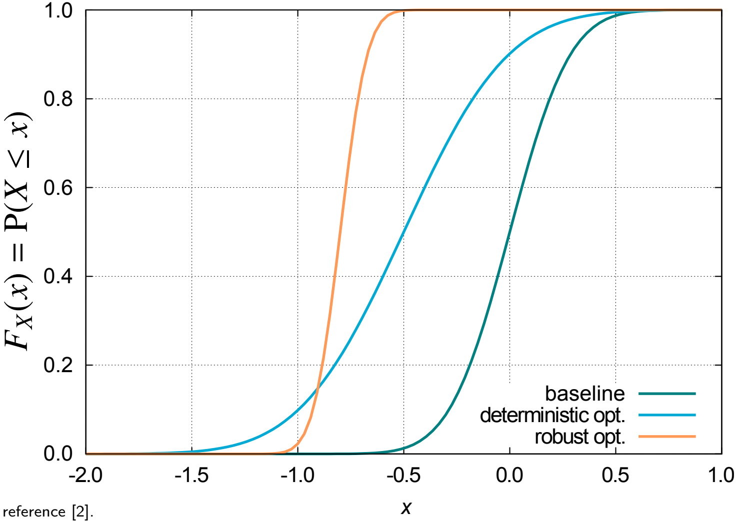
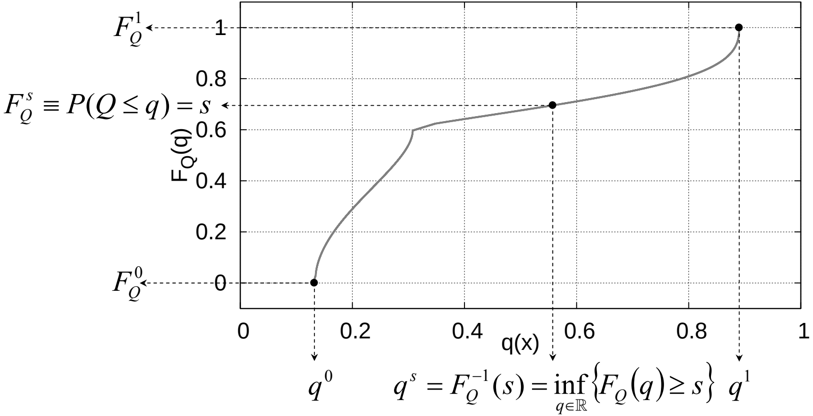


Figure from reference [2].

CDF gives the area under the probability density function from minus infinity to x.

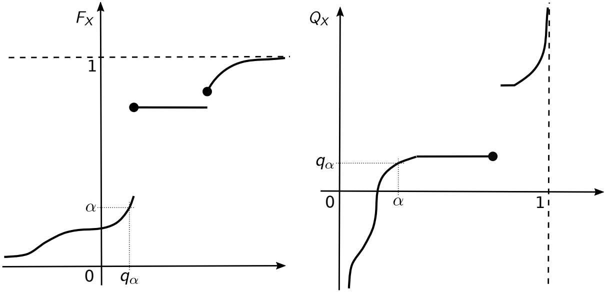
$F_X(x) = \mathrm{P}(X \leq x)$, is the probability that X takes on a value $\leq x$.

Generalized inverse distribution function



CDF and ICDF characteristic points

F_X and its inverse Q_X

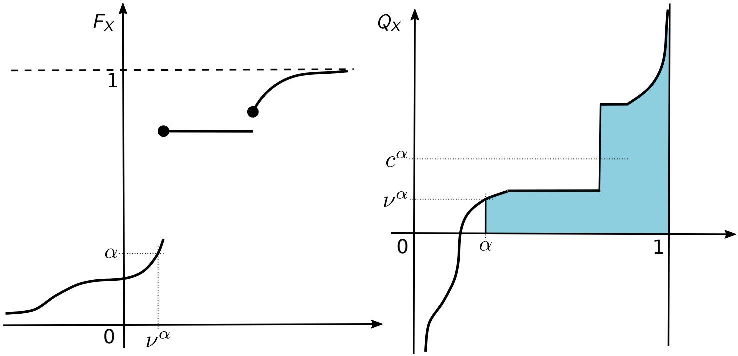


The inverse quantile or Value-at-Risk (VaR) is one of the risk measures adopted here, and it can be easily obtained once if the CDF is available. Let X a random variable and $F(y) = \Pr\{X \leq y\}$ the CDF of X. Then the inverse CDF of X can be defined as $F^{-1}(\gamma) = \inf\{y : F(y) \geq \gamma\}$. For any $\alpha \in (0, 1)$, the α-VaR of X is defined as

$$\nu^\alpha = F^{-1}(\alpha),$$

It is the maximum loss that can be exceeded only in $(1 - \alpha)100\%$ of cases.

Quantile and α-tail



The α-CVaR of X can be thought of as the conditional expectation of losses that exceed the q_α level, and can be expressed as

$$c^\alpha = \frac{1}{1-\alpha}\int_\alpha^1 \nu^\beta \mathrm{d}\beta$$

Quantile (VaR) estimation using the ECDF

If $x_1, x_2, \ldots, x_n$ are n independent and identically distributed (i.i.d.) observations of the random variable X, then the α-VaR of X can be estimated by

$$\hat{\nu}^{\alpha;n} = X_{\lceil n\alpha \rceil : n} = \hat{F}_n^{-1}(\alpha)$$

where $X_{i:n}$ is the i-th order statistic from the n observations, and

$$F_n(t) = \sum_{i=1}^{n} \mathbb{1}\{x_i \leq t\}$$

is the empirical CDF constructed from the sequence $\tilde{X}$ of $x_1, x_2, \ldots, x_n$, $\mathbb{1}\{\cdot\}$ is the indicator function and t is a scalar value

Superquantile (CVaR) estimation using the ECDF I

c^α can also be written as the following stochastic program:

$$c^\alpha = \inf_{t \in \mathbb{R}} \left\{ t + \frac{1}{1-\alpha} E[X-t]^+ \right\}$$

with $[a]^+ = \max\{0, a\}$. The set of optimal solutions to the stochastic program is $T = [\nu^\alpha, u^\alpha]$ with $u^\alpha = \sup t : F(t) \leq \alpha$. In particular, $\nu^\alpha \in T$, so

$$c^\alpha = \nu^\alpha + \frac{1}{1-\alpha} E[X - \nu^\alpha]^+$$

When X has a positive density in the neighborhood of ν^α, then $\nu^\alpha = u^\alpha$. Therefore, the stochastic program has a unique solution, and

$$c^\alpha = E[X | X \geq \nu^\alpha]$$

with $E[X|X \geq \nu^\alpha]$ also known as expected shortfall or tail conditional expectation.

Superquantile (CVaR) estimation using the ECDF II

In the case where we have a finite number of samples, that is a ECDF, and $X_1, X_2, \ldots, X_n$ are n independent and identically distributed (i.i.d.) observations of the random variable X, then the estimator

$$\hat{c}^{\alpha;n} = \inf_{t \in \mathbb{R}} \left\{ t + \frac{1}{n(1-\alpha)} \sum_{i=1}^{n} [X_i - t]^+ \right\}$$

is used to estimate the α-CVaR of L. So we easily obtain the direct estimate of c^α:

$$\hat{c}^{\alpha;n} = \hat{\nu}^{\alpha;n} + \frac{1}{n(1-\alpha)} \sum_{i=1}^{n} [X_i - \hat{\nu}^{\alpha;n}]^+$$

Roadmap of the optimization experiments

1. A deterministic GA optimization is performed at first to find an approximation of the global optimum solution. The choice of using a genetic algorithm is motivated by its high success rate in solving similar problems in the recent past;
2. Once a deterministic solution is found, uncertainty is plugged in and optimization under uncertainty is performed to refine the solution and improve the response probability tail performances. In this phase, both the design and the uncertain variables are active: the aim is to search locally for a robust design solution around the deterministic optimum. Two algorithms are used for this purpose: the SBLO and the CMA-ES.

The GA optimizer minimizes the G function (quantity of interest)

Deterministic GA parameters

Parameter	Value
Pop. size	48
Generation no.	100
Bit mutation rate	1%
One-point crossover rate	80%

Deterministic optimization history

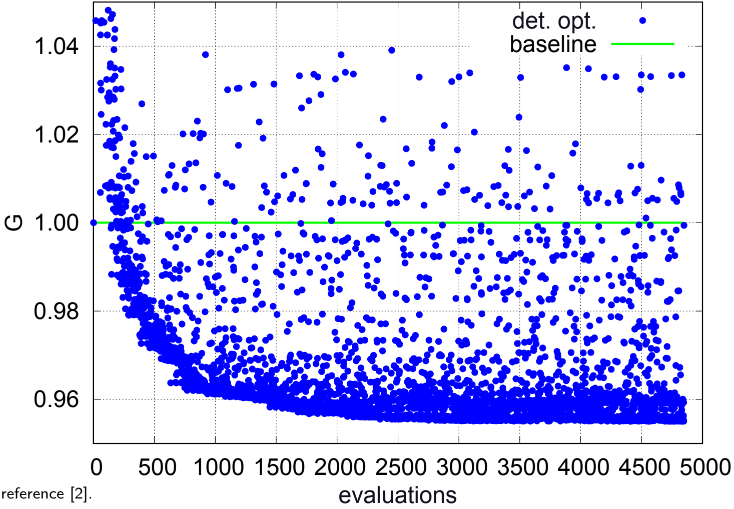


Figure from reference [2].

Evolution history after 100 generations

Comparison of VaR and CVaR curves for different sampling sizes

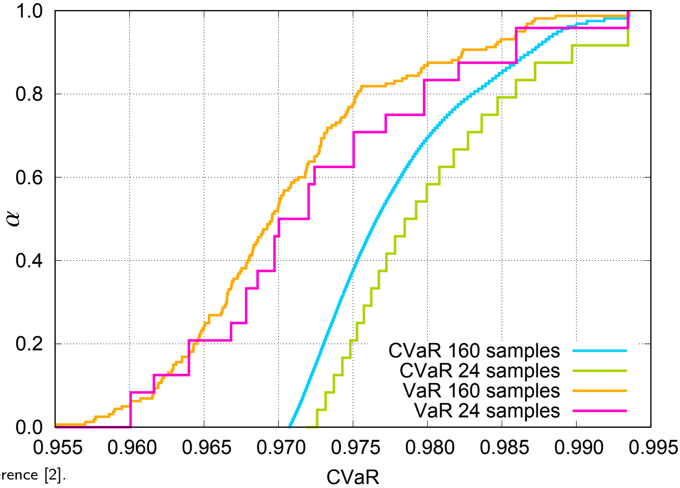


Figure from reference [2].

- ECDF are estimated with just 24 samples during optimization
- ECDF are obtained using 160 samples in the result analysis phase

Surrogate-based local optimization

- SBLO starts with 24 Latin Hypercube samples in the inner loop for estimating VaR at $\alpha = 0.9$. The outer loop employs a Latin Hypercube sampling of size 48 to feed the surrogate model.
- A Kriging surrogate is used with Gaussian correlation functions, reduced quadratic (i.e., mixed terms are neglected) trend function and hyper-parameters tuning.
- The optimization on the surrogate in the trust region is carried out by means of an evolutionary algorithm.
    - A population of 200 individuals is evolved with 100% two-point crossover rate and 10% normal offset mutation rate. A total number of 20,000 fitness function evaluations is fixed.
    - The new population is created by introducing 10 best individuals from the combination of the current population and the newly generated individuals and randomly selecting the remaining 190 individuals among the remaining individuals.

VaR SBLO optimization run

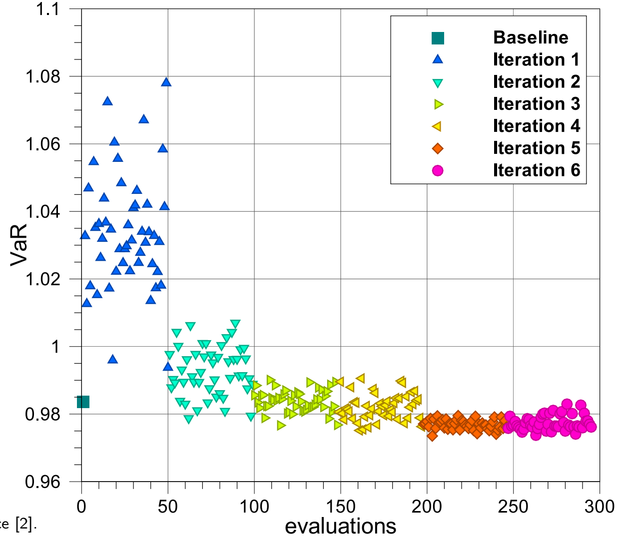


Figure from reference [2].

Evolution history after 33 generations (661 evaluations).

CVaR CMA-ES optimization run

Conditional-Value-at-Risk response function at $\alpha = 0.9$ is considered as the objective function and it is estimated with 24 Monte Carlo samples during the optimization, while in the post-run analysis 160 samples are used for validation.

CMA-ES parameters

Parameter	Value
Initial σ	0.05
λ	20
μ	10
Max. obj. Evaluations	660
Max. iterations	33
Damping for σ	0.746419

VaR + SBLO results

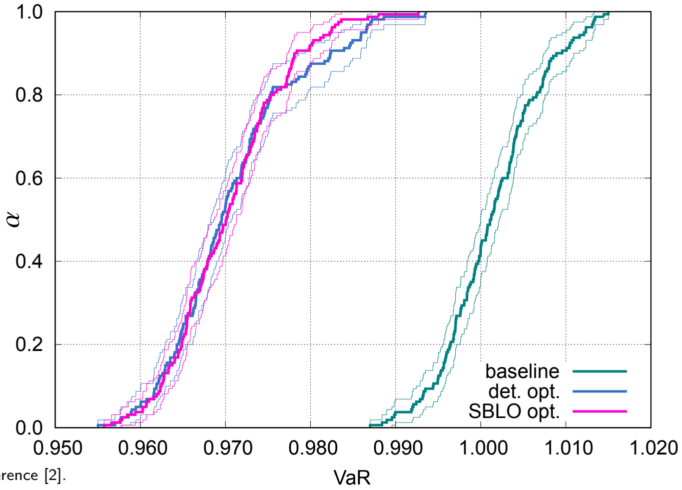


Figure from reference [2].

Comparison of VaR curves with bootstrap c. i. for baseline, deterministic optimum and VaR-based optimum with SBLO

VaR + SBLO results

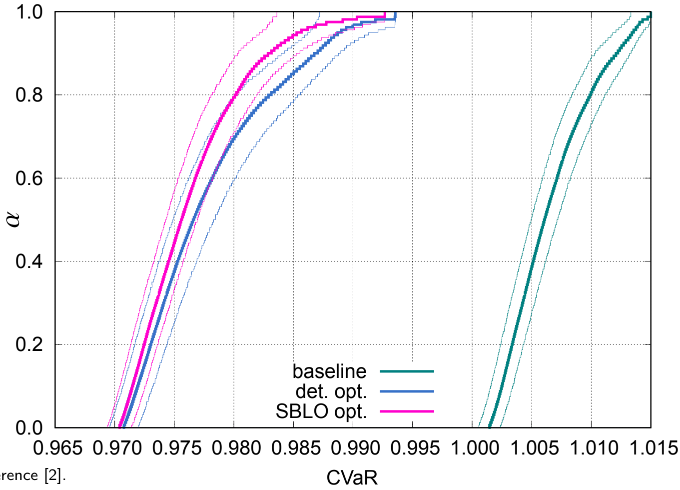


Figure from reference [2].

Comparison of CVaR curves with bootstrap c. i. for baseline, deterministic optimum and VaR-based optimum with SBLO

CVaR + CMA-ES results

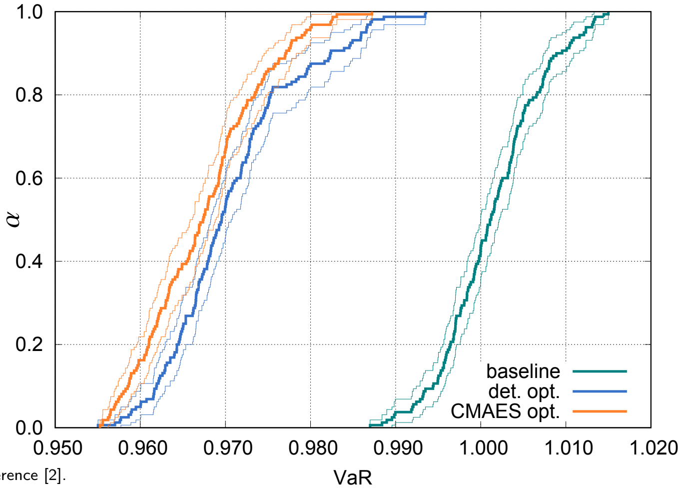
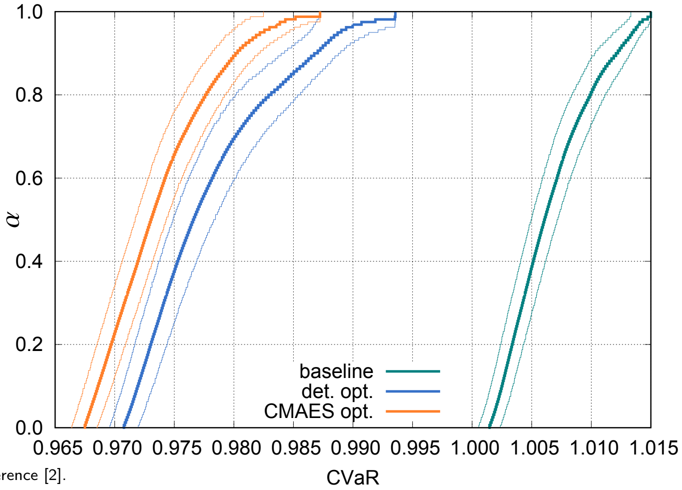


Figure from reference [2].

Comparison of VaR curves with bootstrap c. i. for baseline, deterministic optimum and CVaR-based optimum with CMA-ES

CVaR + CMA-ES results



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Comparison of CVaR curves with bootstrap c. i. for baseline, deterministic optimum and CVaR-based optimum with CMA-ES

Transition line envelopes for the wing upper surface

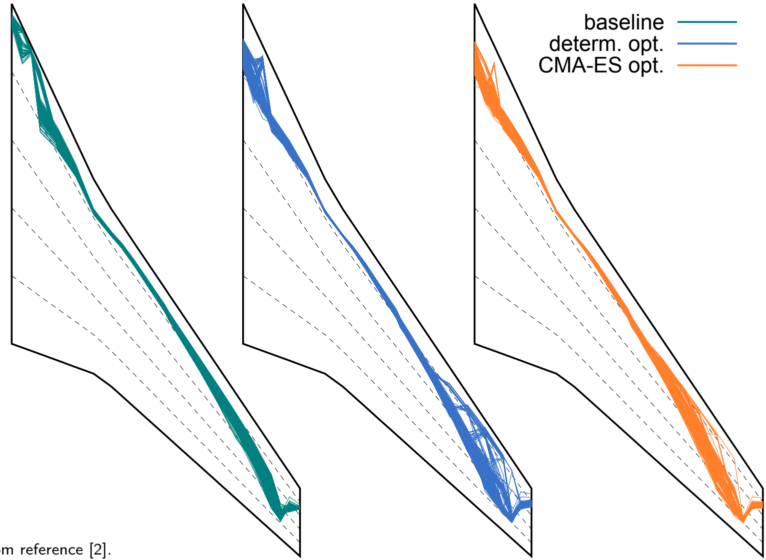

Figure from reference [2].

Transition line envelopes for the wing lower surface

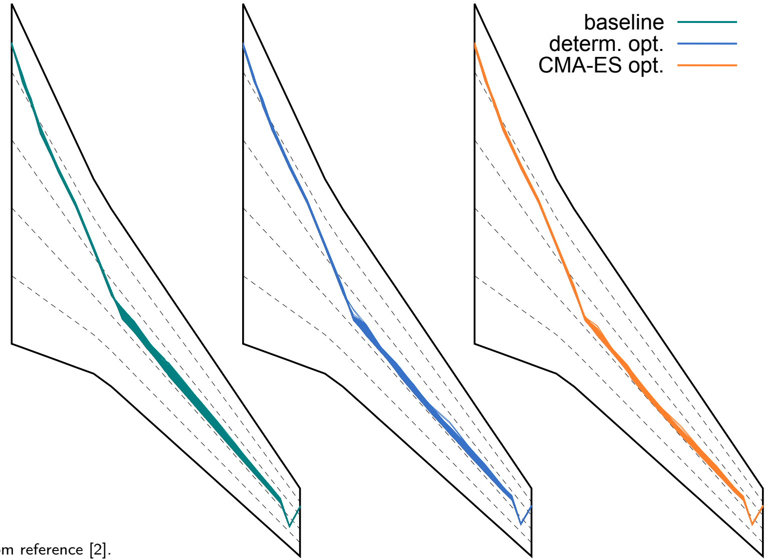

Figure from reference [2].

Conclusions I

- The use of an optimization approach based on VaR and CVaR risk measures has been successfully employed to solve an aerodynamic design problem of industrial interest.
- The high computational power required by the evaluation of the objective function has imposed the use of a very rough estimate of VaR in the optimization process.
- The bootstrap analysis, however, has allowed to verify that, despite the noise produced by the roughness of the estimate, the estimated value of VaR and CVaR were fairly stable and consistent.
- The baseline and optimal solutions obtained in several generations were then validated using a more refined sampling which, together with bootstrap analysis for the calculation of confidence intervals, confirmed the reliability of the solutions obtained through the optimization process.

Conclusions II

- Further work is needed to enhance the ranking capabilities of the algorithm near the optimum, as the noise introduced by the coarseness of the samples could lead to misleading conclusions.
- Therefore, it is envisaged to introduce a hierarchy of computational models of increasing complexity and fidelity as well as advanced statistical sampling techniques such as importance sampling and multilevel Monte Carlo methods.

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